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Optimization of Rectangular and Box Sections in Oblique Bending and Eccentric Compression

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Abstract. The article presents a solution to the problem of finding the optimal ratio of the height of the cross-section to the width for a rectangular and box-shaped section in the case of oblique bending and eccentric compression. Optimization is performed according to the strength criterion, and for the case of oblique bending of a rectangular beam, a solution was also obtained from the condition of a minimum full deflection. For a rectangular section, the solution is made analytically, and for a box section, numerically using the MATLAB environment and the Optimization Toolbox package. As a numerical method of nonlinear optimization, the interior point method is used. To simplify the solution, the box section is assumed to be thin-walled, i.e. it is assumed that the wall thickness is significantly less than the height and width of the cross section. An estimate of the error of such an assumption is also performed. It has been established that in the case of oblique bending of a rectangular beam, when optimizing according to the strength criterion, the optimal ratio of the cross-sectional height to width is equal to the cotangent of the angle between the force plane and the vertical axis, and when optimizing according to the rigidity criterion, it is the square root of the cotangent of this angle. In the case of eccentric compression of a rectangular beam with eccentricities in two planes, the optimal ratio of the height of the cross section to the width is equal to the ratio of the eccentricity along the vertical and horizontal axes. For a box-shaped section, graphs of the change in optimal parameters depending on the angle between the force plane and the vertical axis in the case of oblique bending, as well as depending on the ratio of eccentricities along the axes in the case of eccentric compression, are plotted.
Keywords: optimization, oblique bending, eccentric compression, strength, rigidity, rectangular section, box section

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1. INTRODUCTION

The task of finding the optimal forms of the bars cross sections always been important, since the bar elements are widely used in load-bearing structures. The first studies on this topic were based on analytical approaches, and later on, numerical and numerical-analytical methods began to be used to solve this problem [1-3].

In the literature, there are a large number of solutions to optimization problems for bar elements. The main optimization method is the creation of a variable cross section along the length of the bar. This method is used, for example, in [4-10] and others, while the configuration of the cross section does not vary.

As for the search for optimal ratios of the cross section dimensions and its shape, there are relatively few such works. The article [11] considers the problem of two-criteria optimization of monosymmetric sections of an open profile with pure bending. The optimization criteria are the cross-sectional area and deflection of the structure. The problem of two-criteria optimization is also presented in [12]. In this article, in addition to bending moments in one plane, the action of axial forces is taken into account.

In [13], the efficiency of various monosymmetric sections in pure bending is compared, including I-beam, box-shaped, octagonal and other more complex ones. A similar problem is posed in [14], however, there, the stiffness not only in bending, but also in torsion, acts as optimization criteria. In [15], the optimization of the cross sections of cold-formed steel profiles is carried out from the condition of maximum dissipation of the strain energy during bending in one plane. For optimization, the particle swarm method is used in combination with the finite element method. The paper [16] presents the first successful application of topological optimization to the design of beams cross sections. In articles [17-18], the optimization of the cross-sectional geometry of the chords of trihedral lattice supports made of thin-walled profiles in the form of a pentagon is carried out according to the stability criterion. The pentagonal profile is also considered as a section of truss chords, and for the purpose of using it in such structures, its geometry is optimized in [19-20]. The article [21] is devoted to the optimization of cold-formed thin-walled channels with an open or closed profile of hinged shelves experiencing pure bending. The solution is carried out analytically, restrictions on strength, as well as general and local stability are introduced.

The review shows that in most publications, when solving optimization problems, the simplest types of bar deformations are considered, mainly bending in one plane, and the cases of complex resistance are practically not affected. An urgent task is to develop methods for calculating and optimizing elements in more complex cases of a stress state [22-24]. The work [25] presents a solution to the problem of optimizing a rectangular cross section of a bar under oblique bending from the strength condition. Such cross-sectional dimensions b and h are found at which the stresses do not exceed the design strenth R at the minimum cross-sectional area. In some cases, the limiting condition, on the basis of which the final dimensions of the cross section are determined, is not the condition of strength, but the condition of rigidity. It is also obvious that a rectangular section in terms of material consumption is not the most optimal for oblique bending. Less material consumption can be achieved by using a box section [26-27]. In addition to the action of bending moments in two planes, rectangular and box-section elements can experience the action of axial forces (the case of eccentric compression with eccentricities in two planes). This article discusses the solution of optimization problems for a rectangular and box-shaped section in two cases of complex resistance: oblique bending and eccentric compression with eccentricities along two axes.

2. METHODS AND MATERIALS

For a rectangular section, the optimization problem can be solved analytically. Consideration will begin with the case of an oblique bending of a bar (Fig. 1).

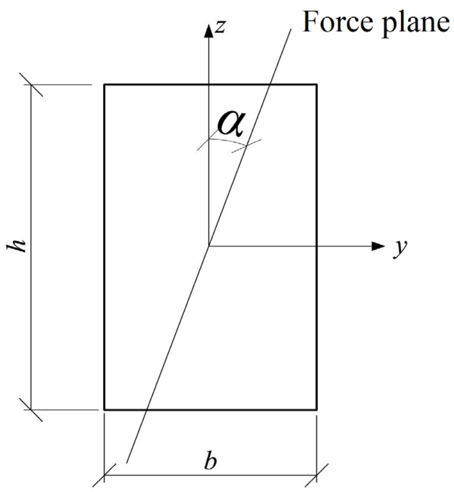

Fig. 1. Rectangular cross-section of a bar undergoing oblique bending.

Let us first perform optimization from the condition of minimum deflection. The material is assumed to be elastic, equally working in tension and compression. Let the plane of action of the load form an angle α with the *z* axis. If, in a straight bending, the maximum deflection is determined as

$$w_{max} = \frac{k}{EI}, \tag{1}$$

where *EI* is the rigidity in the plane of bending, *k* is a certain coefficient depending on the magnitude of the load and span, then in case of oblique bending the components of the total deflection along the *y* and *z* axes will be calculated as follows:

$$f_y = \frac{k \sin\alpha}{EI_z};$$
$$f_z = \frac{k \cos\alpha}{EI_y}. \tag{2}$$

Full deflection is defined as:

$$f = \sqrt{f_y^2 + f_z^2} = \frac{k}{E}\sqrt{\left(\frac{\cos\alpha}{I_y}\right)^2 + \left(\frac{\sin\alpha}{I_z}\right)^2}. \tag{3}$$

The optimization problem is set as follows. It is required to find such a ratio between the dimensions of the cross section *b* and *h*, so that at a constant consumption of material, i.e. constant area *A*, the beam deflection is minimal.

To solve the problem, the moments of inertia are expressed in terms of *A* and *b*:

$$I_y = \frac{bh^3}{12} = \frac{A^3}{12b^2};$$
$$I_z = \frac{b^3 h}{12} = \frac{b^2 A}{12}. \tag{4}$$

Substitution of (4) into (3) leads to the following expression:

$$f=\frac{12k}{EA}\sqrt{\left(\frac{\cos\alpha\cdot b^2}{A^2}\right)^2+\left(\frac{\sin\alpha}{b^2}\right)^2}. \tag{5}$$

To find the minimum point of this function, it is differentiated with respect to b and the derivative is equated to zero:

$$\frac{df}{db}=\frac{24k}{EA}\frac{\left(-\frac{\sin^2\alpha}{b^5}+\frac{b^3\cos^2\alpha}{A^4}\right)}{\sqrt{\frac{\sin^2\alpha}{b^4}+\frac{b^4\cos^2\alpha}{A^4}}}=0;$$
$$-\frac{\sin^2\alpha}{b^5}+\frac{b^3\cos^2\alpha}{A^4}=0; \tag{6}$$
$$b^8=\frac{A^4\sin^2\alpha}{\cos^2\alpha},$$

from which is follows

$$b=\sqrt{A}\sqrt[4]{\frac{\sin\alpha}{\cos\alpha}}. \tag{7}$$

Then

$$h=\frac{A}{b}=\sqrt{A}\sqrt[4]{\frac{\cos\alpha}{\sin\alpha}}, \tag{8}$$

and the optimal ratio h/b will be equal to

$$\frac{h}{b}=\sqrt{ctg\alpha}. \tag{9}$$

If it is required to find the optimal dimensions of the cross section h and b for a given allowable deflection $[f]$, then this can be done as follows. Substituting (7) and (8) into (5), we get:

$$f=\frac{12\sqrt{2}k}{EA^2}\sqrt{\sin\alpha\cos\alpha}\le[f]. \tag{10}$$

From (10), the required cross-sectional area is determined as:

$$A=\sqrt{\frac{12\sqrt{2}k}{E[f]}\sqrt{\sin\alpha\cos\alpha}}. \tag{11}$$

Knowing A, one can find b and h using formulas (7)-(8).

In a similar way, optimization from the strength condition can be performed. Let us pose the optimization problem somewhat differently than in [25]. Let us find such a ratio between the dimensions b and h in order to minimize the greatest stress in the cross section at a constant material consumption.

$$\sigma_{max}=\frac{M_y}{W_y}+\frac{M_z}{W_z}\to min, \tag{12}$$

where M_y and M_z are bending moments about the axes y and z, W_y and W_z are the corresponding moments of resistance..

Moments M_y and M_z in formula (12) are assumed to be positive.

The bending moments M_y and M_z are represented as follows:

$$M_y = M\cos\alpha;$$
$$M_z = M\sin\alpha, \tag{13}$$

where $M = \sqrt{M_y^2 + M_z^2}$.

The moments of resistance are expressed in terms of b and A:

$$W_y = \frac{bh^2}{6} = \frac{A^2}{6b}; W_z = \frac{b^2 h}{6} = \frac{bA}{6}. \tag{14}$$

Substituting (14) and (13) into (12), after minor transformations, one can get:

$$\sigma_{max} = \frac{6M}{A}\left(\frac{\cos\alpha \cdot b}{A} + \frac{\sin\alpha}{b}\right). \tag{15}$$

Let us take the derivative of (15) with respect to b and equate to zero:

$$\frac{d\sigma_{max}}{db} = \frac{6M}{A}\left(\frac{\cos\alpha}{A} - \frac{\sin\alpha}{b^2}\right) = 0. \tag{16}$$

From (16) it follows:

$$b = \sqrt{A\frac{\sin\alpha}{\cos\alpha}};$$
$$h = \frac{A}{b} = \sqrt{A\frac{\cos\alpha}{\sin\alpha}}; \tag{17}$$
$$\frac{h}{b} = \frac{\cos\alpha}{\sin\alpha} = \frac{M_y}{M_z}.$$

To determine the optimal dimensions of the cross section for a given design strength R, expression (17) is substituted into (15). As a result:

$$\sigma_{max} = \frac{12M\sqrt{\sin\alpha\cos\alpha}}{A^{\frac{3}{2}}} = \frac{12\sqrt{M_y M_z}}{A^{\frac{3}{2}}} = R, \tag{18}$$

from which it follows

$$A = \left(\frac{12\sqrt{M_y M_z}}{R}\right)^{\frac{2}{3}}. \tag{19}$$

Next, one can find the dimensions b and h:

$$b = \left(\frac{12}{R}\right)^{\frac{1}{3}} M_y^{-\frac{1}{3}} M_z^{\frac{2}{3}};$$
$$h = \left(\frac{12}{R}\right)^{\frac{1}{3}} M_y^{\frac{2}{3}} M_z^{-\frac{1}{3}}. \tag{20}$$

The obtained formulas (20) coincide with the solution presented in [25].

Similarly, optimization of the rectangular cross section dimensions under eccentric compression can be performed.

The calculation scheme of an eccentrically compressed element is shown in Fig. 2.

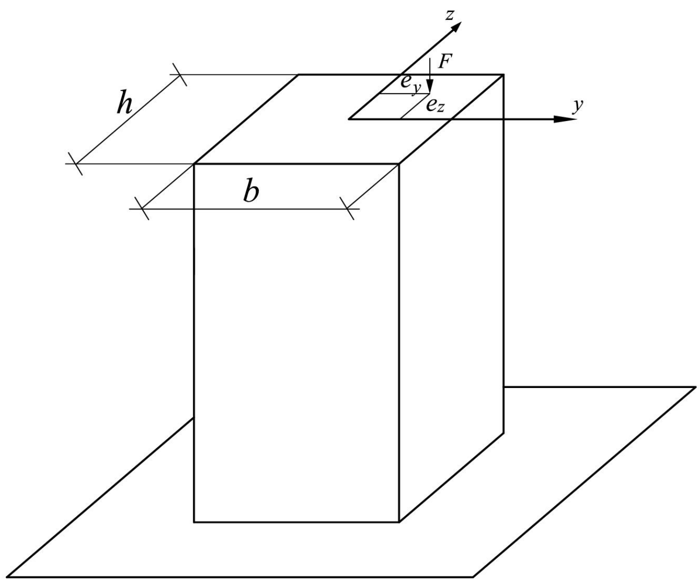

Fig. 2. Calculation scheme of an eccentrically compressed element of a rectangular cross section.

As before, it is assumed that the material resists both tension and compression equally. The calculation is carried out according to a non-deformable scheme, assuming that the deflection of the element does not lead to a noticeable increase in bending moments. Optimization is performed from the strength condition. The objective function is the maximum stress in absolute value, which is calculated as follows:

$$\left|\sigma_{max}\right| = \frac{M_y}{W_y} + \frac{M_z}{W_z} + \frac{F}{A} \rightarrow min. \tag{21}$$

The bending moments M_y and M_z are written as:

$$\begin{aligned} M_y &= F \cdot e_z; \\ M_z &= F \cdot e_y. \end{aligned} \tag{22}$$

Taking into account (22) and (14), formula (21) takes the form:

$$\left|\sigma_{max}\right| = \frac{6F}{A}\left(\frac{e_z b}{A} + \frac{e_y}{b}\right) + \frac{F}{A}. \tag{23}$$

Differentiating (23) with respect to b and equating to zero, we get:

$$\frac{d\left|\sigma_{max}\right|}{db} = \frac{6F}{A}\left(\frac{e_z}{A} - \frac{e_y}{b^2}\right) = 0, \tag{24}$$

from which it follows

$$b = \sqrt{A\frac{e_y}{e_z}};$$
$$h = \frac{A}{b} = \sqrt{A\frac{e_z}{e_y}}. \tag{25}$$

The optimal ratio *h/b* turns out to be equal to the ratio of eccentricities:

$$\frac{h}{b} = \frac{e_z}{e_y}. \tag{26}$$

The obtained solution is also valid for the case of off-center tension.

To find the dimensions of the cross section for a given design strength, one should substitute (25) into (23):

$$\left|\sigma_{max}\right| = \frac{12F\sqrt{e_y e_z}}{A^{\frac{3}{2}}} + \frac{F}{A} = R. \tag{27}$$

Let us multiply (27) by $A^{3/2}$ and introduce the variable $t = \sqrt{A}$. Then the problem of determining the optimal dimensions of the cross section for a given design strength will be reduced to a cubic equation for *t*:

$$Rt^3 = 12F\sqrt{e_y e_z} + Ft. \tag{28}$$

Next, the minimum positive root of this equation is found, then the cross-sectional area $A = t^2$ and the dimensions *b* and *h* are calculated according to formulas (25).

Let us further consider the technique for optimizing the box section (Fig. 3).

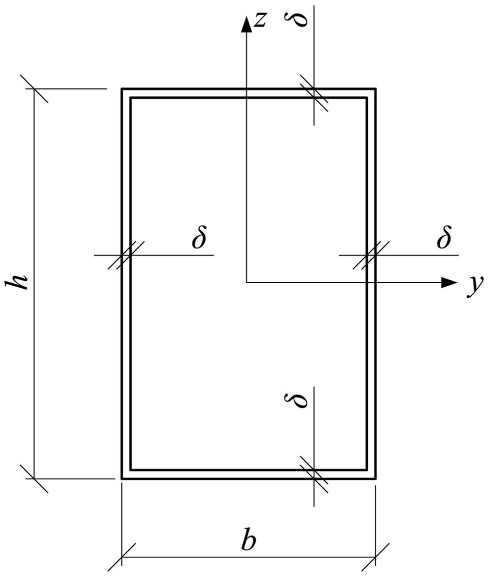

Fig. 3. Box cross section.

Let us find such a ratio between the dimensions h and b, which ensures the maximum load-bearing capacity at a constant material consumption. For simplicity, it will be assumed that the section is thin-walled, i.e. $h \gg \delta$ and $b \gg \delta$.

The cross-sectional area will be written as:

$$A = 2(b+h)\delta. \tag{29}$$

Axial moments of inertia are calculated as follows:

$$I_y = 2\left(\frac{h^3\delta}{12} + \frac{b\delta h^2}{4}\right);$$
$$I_z = 2\left(\frac{b^3\delta}{12} + \frac{h\delta b^2}{4}\right). \tag{30}$$

The moments of resistance are written as:

$$W_y = \frac{2I_y}{h} = 4\left(\frac{h^2\delta}{12} + \frac{b\delta h}{4}\right) = \delta h\left(\frac{h}{3} + b\right);$$
$$W_z = \frac{2I_z}{b} = 4\left(\frac{b^2\delta}{12} + \frac{h\delta b}{4}\right) = \delta b\left(\frac{b}{3} + h\right). \tag{31}$$

The maximum stress, as before, is determined by formula (12).
The moments of resistance are expressed in terms of A, δ and b:

$$h = \frac{A}{2\delta} - b \to W_y = \delta\left(\frac{A}{2\delta} - b\right)\left(\frac{A}{6\delta} + \frac{2}{3}b\right);$$
$$W_z = \delta b\left(\frac{A}{2\delta} - \frac{2}{3}b\right). \tag{32}$$

Then the formula for maximum stress will take the form:

$$\sigma_{max} = \frac{M}{\delta}\left(\frac{\cos\alpha}{\left(\frac{A}{2\delta} - b\right)\left(\frac{A}{6\delta} + \frac{2}{3}b\right)} + \frac{\sin\alpha}{b\left(\frac{A}{2\delta} - \frac{2}{3}b\right)}\right). \tag{33}$$

Let us introduce the quantities $\frac{A}{2\delta} = p$ – the semiperimeter of the section and $x = b/p$, $x \in (0;1)$.
Then formula (33) takes the form:

$$\sigma_{max} = \frac{M}{\delta p^2}\left(\frac{\cos\alpha}{(1-x)\left(\frac{1}{3} + \frac{2}{3}x\right)} + \frac{\sin\alpha}{x\left(1 - \frac{2}{3}x\right)}\right). \tag{34}$$

The results of the search for the minimum of function (34) will be given further in the next section. To find the minimum point, the Optimization Toolbox package of the MATLAB environment was used. The nonlinear optimization problem was solved using the interior point method [28, 29].

In the case of eccentric compression, the maximum modulus stress, when considering a box-shaped section as a thin-walled one, will be written in the form:

$$|\sigma_{max}| = \frac{Fe_z}{\delta\left(\frac{A}{2\delta} - b\right)\left(\frac{A}{6\delta} + \frac{2}{3}b\right)} + \frac{Fe_y}{\delta b\left(\frac{A}{2\delta} - \frac{2}{3}b\right)} + \frac{F}{A} =$$
$$= \frac{F}{\delta p}\left(\frac{e_z / p}{(1-x)\left(\frac{1}{3} + \frac{2}{3}x\right)} + \frac{e_y / p}{x\left(1 - \frac{2}{3}x\right)} + \frac{1}{2}\right). \tag{35}$$

The multiplier in front of the bracket at a constant material consumption is a constant, so the problem of finding the minimum point of the function $|\sigma_{max}|$ is equivalent to the problem of finding the minimum of the function

$$f_1 = \frac{e_z / p}{(1-x)\left(\frac{1}{3}+\frac{2}{3}x\right)} + \frac{e_y / p}{x\left(1-\frac{2}{3}x\right)} + \frac{1}{2}. \tag{36}$$

In turn, if x_{opt} is the minimum point of the function f_1, it will also be the minimum point of the function

$$f_2 = \frac{f_1 - \frac{1}{2}}{e_z} = \frac{1}{p}\left(\frac{1}{(1-x)\left(\frac{1}{3}+\frac{2}{3}x\right)} + \frac{e_y / e_z}{x\left(1-\frac{2}{3}x\right)}\right). \tag{37}$$

The results of the search for the minimum of the function f_2 will be given in the next section.

3. RESULTS AND DISCUSSION

Figure 4 shows the dependence of the stress σ_{max} at oblique bending on x for various values of α (at M, δ, p = const). Each of the graphs has a clearly defined minimum point x_{opt}. The plot of the dependence of the optimal value x_{opt} on the angle α is shown in Fig. 5. For direct bending in a plane xOz ($\alpha = 0$) the optimal value of x is 0.25, and the corresponding ratio $h/b = 3$. For direct bending in a plane xOy ($\alpha = \pi/2$) $x_{opt} = 0.75$, and $h/b = 1/3$. At $\alpha = \pi/4$ $x_{opt} = 0.5$, and $h = b$. This result agrees with the solution obtained in [26–27]. Note that for a rectangular section, in contrast to a box-shaped one, with a direct bending in the xOz plane, the optimal h tends to infinity, and b tends to zero.

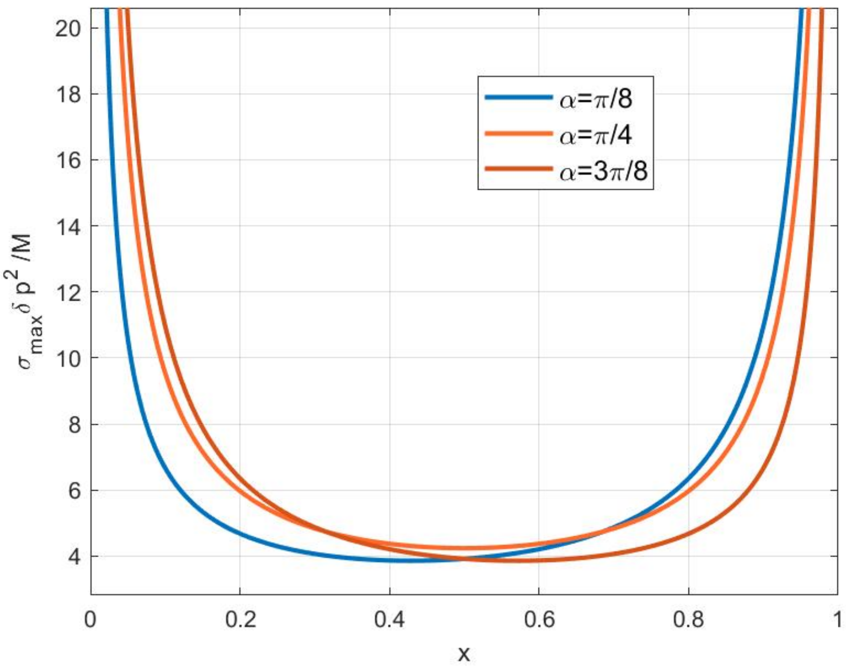

Fig. 4. Dependence of σ_{max} on x for different values of α.

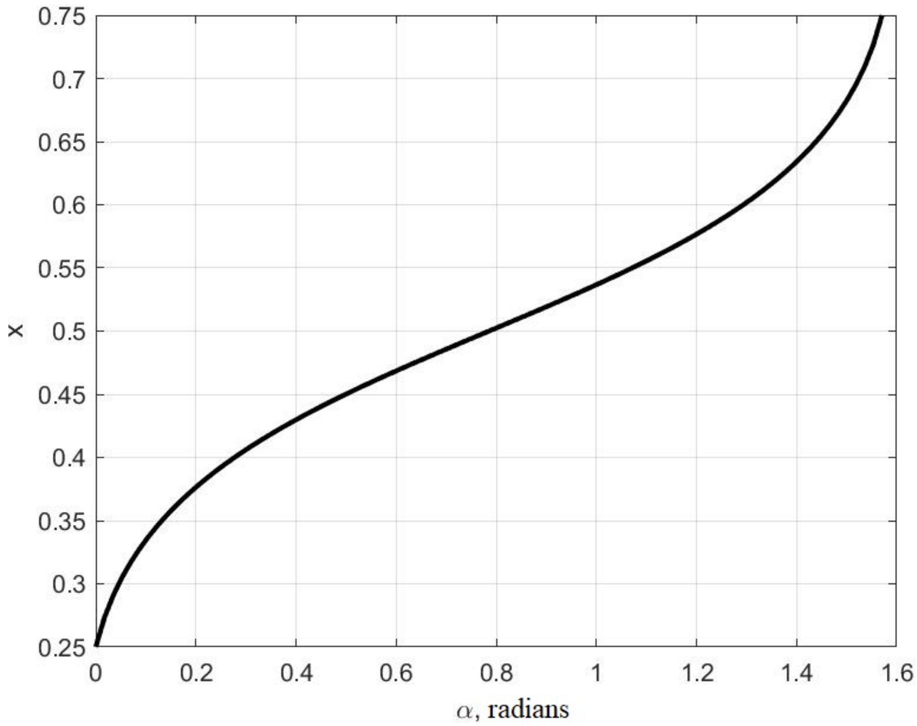

Fig. 5. Dependence of the optimal value x_{opt} on the angle α.

Also, to analyze the influence of the wall thickness on the optimal value of x, the problem was solved taking into account the real dimensions of the cross section. For this, the axial moments of inertia were calculated by the formulas:

$$I_y = \frac{bh^3}{12} - \frac{(b-2\delta)(h-2\delta)^3}{12};$$
$$I_z = \frac{b^3 h}{12} - \frac{(b-2\delta)^3 (h-2\delta)}{12}. \qquad (38)$$

The calculation was carried out at $\delta = 4$ mm, $p = 200$ mm (at $b = h$ such dimensions correspond to a square pipe 100x100x4). Table 1 shows a comparison of the optimal values of x, obtained without taking into account and taking into account the wall thickness.

Table 1. Optimal x values obtained without and with taking into account the wall thickness.

α, degrees		0	10	20	30	40	45
x_{opt}	without taking into account the thickness	0.25	0.3674	0.4183	0.4546	0.4854	0.5
	taking into account the thickness	0.2507	0.3718	0.4213	0.4564	0.4860	0.5

Table. 1 shows that the results differ slightly. Thus, it can be concluded that the optimal ratio $\frac{h}{b} = \frac{1-x}{x}$ does not depend on the wall thickness.

Fig. 6 shows the graphs of the dependence of the function $f_2(x)$ that determines the maximum stresses in the box section under eccentric compression for various ratios e_y/e_z.

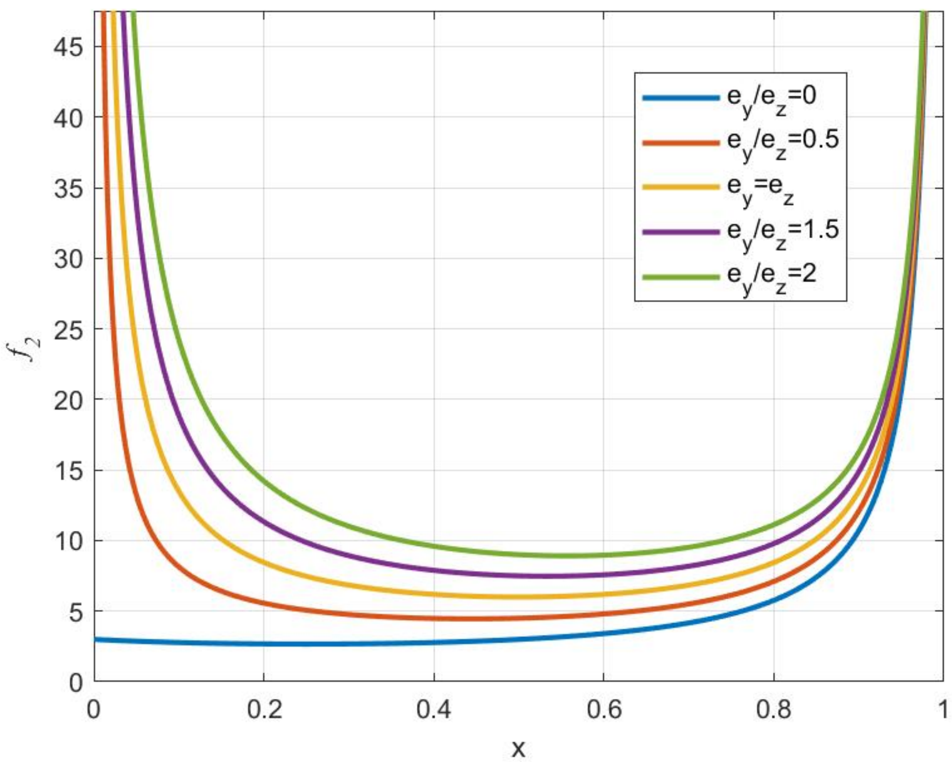

Fig. 6. Plots of $f_2(x)$ for different ratios e_y/e_z.

Fig. 7 shows a plot of the optimal x value versus the e_y/e_z ratio. At the eccentric compression with eccentricity only along the z axis ($e_y/e_z = 0$): $x_{opt} = 0.25$, which corresponds to $h/b = 3$, and in the presence of eccentricity only along the y axis ($e_y/e_z \rightarrow\infty$) x_{opt} asymptotically approaches the value 0.75, which corresponds to the relation $h/b = 1/3$. With the same eccentricities in two planes, $x_{opt} = 0.5$, i.e. $b = h$.

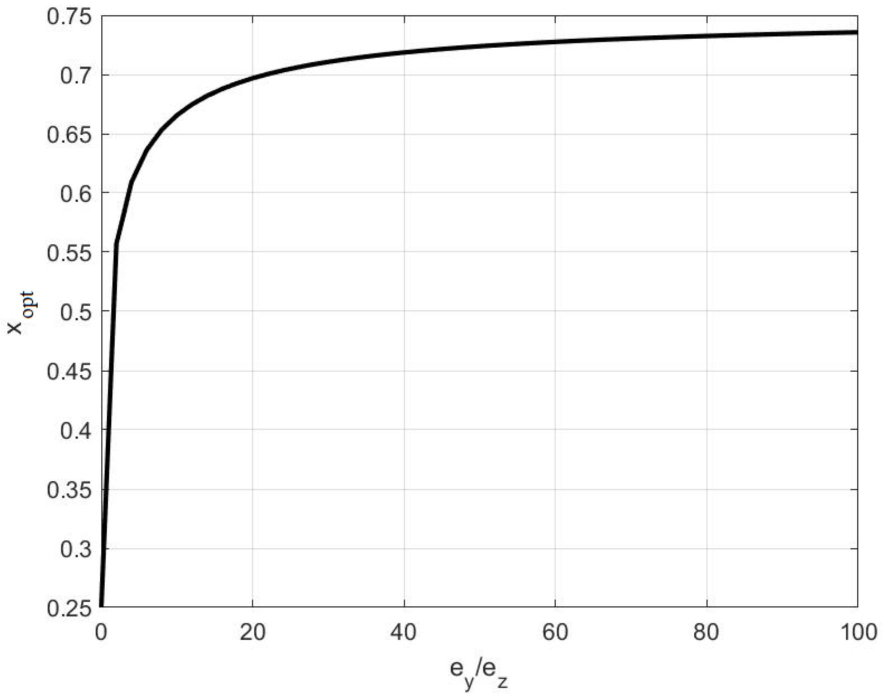

Fig. 7. Dependence of the optimal value x_{opt} on the ratio of eccentricities.

4. CONCLUSIONS

1. The problem of optimizing the dimensions of a rectangular cross section under oblique bending was solved analytically. When optimizing according to the stiffness criterion, the optimal ratio of the height of the cross section to the width *h*/*b* is equal to $\sqrt{ctg\alpha}$, where α is the angle between the vertical axis and the force plane. In the case of optimization according to the strength criterion, the optimal ratio of the height of the cross section to the width is equal to $ctg\alpha$.

2. An analytical solution to the problem of finding the optimal ratio *h*/*b* of a rectangular cross section under off-center compression with eccentricities in two planes is obtained. The optimal ratio *h*/*b* is equal to the ratio of the eccentricities along the *z* and *y* axes.

3. The problem of optimizing the geometry of a box section with oblique bending is solved numerically. It has been established that in the particular case with direct bending (α = 0), the optimal ratio of the height of the box section to the width is equal to 3. The thickness of the wall of the box section does not significantly affect the optimal ratio *h*/*b*.

4. A numerical solution is obtained for the problem of finding the optimal ratio *h*/*b* of a box section under eccentric compression. If there is eccentricity along only one axis, the size of the section along this axis must be 3 times the size along the other axis.

REFERENCES

[1] Mayencourt P., Mueller C. Hybrid analytical and computational optimization methodology for structural shaping: Material-efficient mass timber beams. Engineering Structures. 2020. 215. Article 110532.

[2] Shariat M. et al. Computational Lagrangian Multiplier Method by using for optimization and sensitivity analysis of rectangular reinforced concrete beams. Steel Compos. Struct. 2018. 29 (2). P. 243 – 256.

[3] Villar-García J. R. et al. Cost optimisation of glued laminated timber roof structures using genetic algorithms. Biosystems Engineering. 2019. 187. P. 258 – 277.

[4] Qiao H., Li H. The discussion on optimization models of pure bending beam. International Journal of Advanced Structural Engineering. 2013. 5. P. 1 – 10.

[5] Mukherjee P., Punera D., Mishra M. Coupled flexural torsional analysis and buckling optimization of variable stiffness thin-walled composite beams. Mechanics of Advanced Materials and Structures. 2022. Vol. 29. 19. P. 2795 – 2815.

[6] Sohouli A., Yildiz M., Suleman A. Efficient strategies for reliability-based design optimization of variable stiffness composite structures. Structural and Multidisciplinary Optimization. 2018. 57. P. 689 – 704.

[7] Punera D., Mukherjee P. Recent developments in manufacturing, mechanics, and design optimization of variable stiffness composites. Journal of Reinforced Plastics and Composites. 2022. 41 (23-24). P. 917 – 945.

[8] Hao P. et al. An integrated framework of exact modeling, isogeometric analysis and optimization for variable-stiffness composite panels. Computer Methods in Applied Mechanics and Engineering. 2018. 339. P. 205 – 238.

[9] Zhu G. et al. Design optimisation of composite bumper beam with variable cross-sections for automotive vehicle. International journal of crashworthiness. 2017. 22 (4). P. 365 – 376.

[10] Duan L. et al. Parametric modeling and multiobjective crashworthiness design optimization of a new front longitudinal beam. Structural and Multidisciplinary Optimization. 2019. 59. P. 1789 – 1812.

[11] Kasperska R.J., Magnucki K., Ostwald M. Bicriteria optimization of cold-formed thin-walled beams with monosymmetrical open cross sections under pure bending. Thin-Walled Structures. 2007. 45 (6). P. 563 – 572.

[12] Manevich A.I., Raksha S.V. Two-criteria optimization of H-section bars–beams under bending and compression. Thin-Walled Structures. 2007. 45 (10-11). P. 898 – 901.

[13] Hämäläinen O.P., Björk T. Optimization of the cross-section of a beam subjected to bending load. Design, Fabrication and Economy of Metal Structures: International Conference Proceedings 2013, Miskolc, Hungary, April 24-26, 2013. Springer Berlin Heidelberg, 2013. P. 17 – 22.

[14] Afzal W., Mufti R.A. Optimal cross-section of cross member for increased torsional and bending stiffness of ladder frame chassis. 2019 16th International Bhurban Conference on Applied Sciences and Technology (IBCAST). IEEE, 2019. P. 218 – 228.

[15] Ye J. et al. Development of optimum cold-formed steel sections for maximum energy dissipation in uniaxial bending. Engineering structures. 2018. 161. P. 55 – 67.

[16] Kim Y.Y., Kim T.S. Topology optimization of beam cross sections. International journal of solids and structures. 2000. 37 (3). P. 477 – 493

[17] Chepurnenko A.S. et al. Trihedral lattice towers with optimal cross-sectional shape. IOP Conference Series: Materials Science and Engineering. IOP Publishing, 2021. 1083 (1). Article 012012.

[18] Sabitov L.S., Badertdinov I.R., Chepurnenko A.S. Optimization of the shape of the cross section of the chords of trihedral lattice supports. Construction and architecture. 2019. 7 (4). P. 5 – 8.

[19] Marutyan A.S. Optimization of pentagonal profile pipes of a new modification. Construction mechanics and calculation of structures. 2016. 3. P. 25 – 35.

[20] Marutyan A.S. Bent-closed profiles and calculation of their optimal parameters. Structural Mechanics of Engineering Constructions and Buildings. 2019. 15 (1). P. 33 – 43.

[21] Magnucki K., Paczos P. Theoretical shape optimization of cold-formed thin-walled channel beams with drop flanges in pure bending. Journal of constructional steel research. 2009. Vol. 65 (8-9). P. 1731 – 1737.

[22] Novoselov O.G., Sabitov L.S., Sibgatullin K.E., Sibgatullin E.S., Klyuev A.S., Klyuev S.V., Shorstova E.S. Method for calculating the strength of massive structural elements in the general case of their stress-strain state (kinematic method). Construction Materials and Products. 2023. 6. (3). P. 5 – 17. https://doi.org/10.58224/2618-7183-2023-6-3-5-17

[23] Novoselov O.G., Sabitov L.S., Sibgatullin K.E., Sibgatullin E.S., Klyuev A.V., Klyuev S.V., Shorstova E.S. Method for calculating the strength of massive structural elements in the general case of their stress-strain state (parametric equations of the strength surface). Construction Materials and Products. 2023. 6 (2). P. 104 – 120. https://doi.org/10.58224/2618-7183-2023-6-2-104-120

[24] Shorstov R.A., Yaziev S.B., Chepurnenko A.S., Klyuev A.V. Flat bending shape stability of rectangular cross-section wooden beams when fastening the edge stretched from the bending moment. Construction Materials and Products. 2022. 5 (4). P. 5 – 18. https://doi.org/10.58224/2618-7183-2022-5-4-5-18

[25] Shein A.I. Optimal dimensions of a rectangular section of a bar with an oblique bend //Bulletin of the Eurasian Science. 2016. 8 (2-33). URL: https://naukovedenie.ru/PDF/116TVN216.pdf

[26] Moiseev G.D., Kolesnikov P.G. Minimization of the mass of structural elements of a box-shaped section at oblique bending. Forestry and chemical complexes-problems and solutions. 2016. P. 100 – 103.

[27] Moiseev G.D., Ivannikova E.A., Kolesnikov P.G., Box-section shape of minimum mass at oblique bending. Machines, Aggregates and Processes. Design, creation and modernization. 2018. P. 68 – 69.

[28] Byrd R. H., Hribar M. E., Nocedal J. An interior point algorithm for large-scale nonlinear programming. SIAM Journal on Optimization. 1999. 9 (4). P. 877 – 900.

[29] Ostroukh E. N. et al. On the issue of the effectiveness of methods and algorithms for solving optimization problems, taking into account the specifics of the objective function. Advanced Engineering Research. 2019. 19 (1). P. 81 – 85.

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