








## Probabilistic analysis of the “multilayer soil – structure” system response to seismic load

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**Abstract.** Based on the analytical model of a horizontal layered medium, applying the probabilistic formulation, the article presents the results of the investigation of joint work of a structure and multilayer soil bed subjected to seismic loading. The damping properties of soil were taken into account. The authors drew a comparison between the fundamental frequencies of the free vibrations of the “soil - structure” system obtained using the layered medium model and the platform model. By the example of a two-layer soil bed, the dependence of the resonant frequencies of the system on the thickness of the near-surface or buried weak layer was determined.

The results of the analysis of the “two-layer soil - structure” system for seismic loads at various locations of the weak layer were presented. The seismic acceleration of the soil bed was modeled as a stationary random process with a given spectral density. The investigation included an analysis of the amplitude-frequency characteristics, acceleration spectral densities and dynamic coefficients for both the entire system and the individual layers. It was demonstrated that the resonant frequencies of an individual layer being a part of the multilayer system can differ significantly from the resonant frequencies of a homogeneous soil bed with similar dynamic characteristics. A comparison between the dynamic responses of the two-layer soil bed system and a system with the reduced characteristics of the soil bed was drawn at various parameters of the spectral density of seismic load. The intervals of possible values of the resonant frequencies of the system were determined taking into account the random variability of the velocity of transverse waves within each layer.

**Keywords:** multilayer soil bed, seismic load, layered medium model, random parameters, free vibrations, resonant frequencies, amplitude-frequency characteristic, dynamic coefficient

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## 1. INTRODUCTION

The investigation of soil-structure interaction during earthquakes is one of the urgent problems of the theory of seismic resistance. The intensity of seismic impact on a structure depends on the ratio of the spectra of seismic vibrations of soil and the natural frequencies of the structure, as well as on the conditions of its support on the soil [1, 2, 3]. In [3], an analysis of seismic risk for 20 structures of various structural systems built on the three main types of soil was conducted taking into account such parameters as the height and aspect ratio of the building, the depth of the foundation location, the ratio of the masses of the structure and the foundation, etc. It was found out that the probability of the maximum damage due to an earthquake is determined primarily by the stiffness of the structure and the type of soil. With an increase in the stiffness of the structure and the flexibility of the soil bed, the probability of creating unsafe or uneconomical structures increases.

The engineering-geological conditions of the construction site have a significant influence on the value of the seismic effect. The geological profile, as a rule, is of layered type with different physical and mechanical characteristics of each layer. When a seismic wave passes through soil, it undergoes multiple reflections and refractions during the transition from one layer to another. Modern software packages enable solving problems of joint analysis of a structure and a multilayer soil bed taking into account various properties of the layers including the nonlinear behavior of the soil. However, the use of complex models in applied problems of seismic resistance of structures can lead to an unacceptable accumulation of errors in the course of analysis due to high uncertainty and incompleteness of the initial information. The initial engineering-geological data are always known approximately, and they are characterized by high statistical variability even within one and the same homogeneous layer. Seismic load is a random process the probabilistic parameters of which should be also taken into account when computing the response of the "soil - structure" system.

The investigations presented in the works [4] and [5] show a significant dependence of the natural vibration frequency and, consequently, the response spectra at free earth surface on the random variability of the thickness, velocity and attenuation parameters of the layers of multilayer soil. Seismic acceleration of soil is modeled as a stationary random function. The probabilistic nature of the specified parameters determines the stochastic nature of the natural frequency and the attenuation coefficient of an equivalent oscillator used by the authors in order to model the vibration of the multilayer stratum as a whole. An investigation of the nature of seismic wave propagation in multilayer soil with a random distribution of the shear modulus and a random thickness of each layer [6] shows that taking into account the stochastic heterogeneity of the shear modulus leads to a decrease in the amplitudes of vibrations at the soil surface and a shift in the spectrum to the region of lower frequencies. Taking into account the random nature of the layer thickness leads to an increase in the amplitudes of vibrations and a shift in the spectrum to the high-frequency region.

Therefore, performing the analysis of stochastic systems should be based on the probabilistic principles and followed by an assessment of the reliability (risk) of the resulting solution [3, 7]. In this case, it is reasonable to use simplified analytical models of structures and soil beds and apply analytical methods for their probabilistic analysis together with the Monte Carlo method [8-11].

In engineering practice, to take into account the soil flexibility, a platform model is used which has a form of a round or rectangular stamp to which, in the general case, six pairs of springs and dampers are attached. These elements simulate the equivalent dynamic stiffness and damping properties of the soil bed. Represented by complex impedance functions of frequency, the equivalent characteristics of soil bed include a real part reflecting the equivalent stiffness, and an imaginary part associated with energy dissipation in the soil bed. These characteristics are determined through solving the problem of stamp oscillations on an elastic homogeneous or layered foundation. A probabilistic solution to the problem of vibrations of the "soil – structure" system based on the platform model is given in the works [11-14].

Using a model of a circular foundation on a random soil layer limited by a homogeneous half-space, it was stated in the article [11] that the stochastic variability of the layer thickness and shear wave velocity with variations of 10% and 20%, respectively, leads to a decrease in the average response of the structure by 39% and, consequently, to an increase in the damping of the coupled system. The horizontal and rocking components of the impedance functions are taken into account.

The paper [12] is devoted to the investigation of the influence of uncertainties in soil parameters (layer thickness and shear wave velocity), as well as in the structural parameters (structure height and foundation radius) on the impedance functions of horizontal and rocking vibrations of the "soil-shallow foundation - structure" system using the Monte Carlo method. Underlain by a homogeneous half-space, a rigid massless circular foundation on the surface of a soil layer with random characteristics is considered.

The article [13] analyzes the seismic response of a rigid massless square foundation on a viscoelastic soil layer bounded by rigid bedrock. Various types of waves (P, SV, SH) incident at an angle or propagating near surface are considered. Impedance matrices were obtained that relate the applied forces with the resulting displacement.

A comparative investigation of nonlinear and stochastic methods for the analysis of the functions of impedance and response of a coupled system was conducted by the authors of the work [14]. The random nature of the shear modulus ( $G$ ) and of the attenuation coefficient ( $\xi$ ) of soil is considered.

The conducted investigations show that the uncertainty of the soil and structure properties significantly affects the seismic behavior of the "soil – structure" system. In the course of seismic interaction of buildings on shallow foundations with a homogeneous soil bed (also in the case when random parameters are taken into account), the main effect includes an extension of the period and an increase in the damping of the "soil - structure" system.

The platform model is applicable only for a homogeneous half-space. The impedance functions depending on the dynamic parameters of the soil bed, the superstructure (a linear oscillator with reduced characteristics) and the foundation geometry allow for a probabilistic analysis of the "soil – foundation - structure" system. At the same time, the surface soil stratum, as a rule, has a multilayer arrangement. As compared to a homogeneous soil bed with the reduced characteristics, various physical, mechanical and geometric parameters of each of the layers, as well as the order of their arrangement, significantly change the dynamic response of the "soil - structure" system.

This article presents the investigation results of the influence of seismic stiffness values of a multilayer soil bed on the dynamic response of the "soil - structure" system based on an analytical computational model of a horizontal layered medium.

A well-known practical application of the problem of vibration of a multilayer soil stratum is the analysis of the degree of intensification or attenuation of seismic impact at the ground surface [4-6,15]. The model of a horizontal layered medium can also be used to assess the interaction of a structure with a multilayer soil bed. The structure is considered as one or more constituent layers of a multilayer system with reduced dynamic characteristics. With this formulation, a single wave pattern arises in the structure and the soil bed under the action of a seismic wave.

This model allows evaluating the special features of resonant processes during joint vibration of a building and the soil bed, as well as taking into account the stochastic nature of seismic load and the soil bed parameters.

## 2. METHODS AND MATERIALS

### Analytical model of "soil - structure" system

The analytical scheme of the "multilayer soil - structure" system is presented in Fig. 1. The system consists of  $n+2$  layers. The zero layer corresponds to the structure,  $n+1$ th layer corresponds to the bedrock.

The characteristics of each of the layers are as follows: thickness  $H_j$ , density  $\rho_j$ , shear modulus  $G_j$ , velocity of transverse elastic wave propagation  $v_j$  ( $j=0, 1, \dots, n+1$ ). The transverse seismic wave  $\tilde{f}(t)$  propagates vertically from the bedrock and is set as a stationary random process. We consider the building (layer 0) as an element of a multilayer system in the form of a shear member with reduced characteristics. We assume that the displacement of points on the line of the foundation bottom coincides with the displacement of points on the upper edge of layer 1.

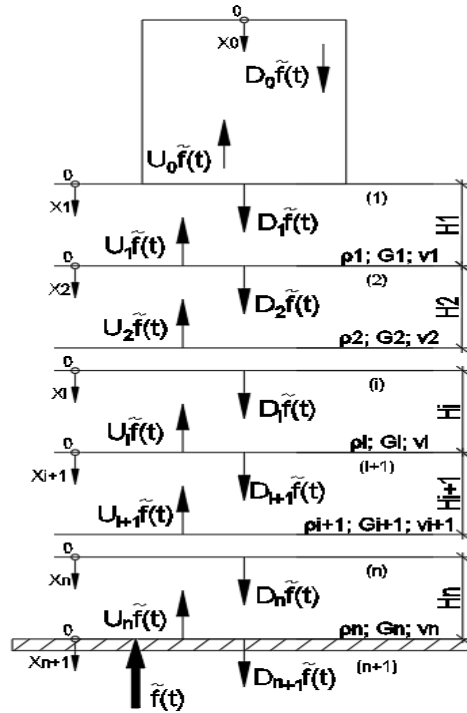


Fig. 1. Analytical layered “soil - structure” model.

Solution methods

The frequencies and modes of free transverse vibrations of a multilayer system can be found from the wave equation solution

$$G_k \frac{\partial^2 u_k(x,t)}{\partial x^2} - \rho_k \frac{\partial^2 u_k(x,t)}{\partial t^2} = 0, \quad H_{k-1} < x \leq H_k, \quad k=1,2,\dots,n, \tag{1}$$

where:  $u_k(x, t)$  – is the horizontal displacement of the layer with coordinate  $x$ .

After separating the variables, we obtain the equation

$$u_k''(x) + \lambda_k^2 u_k(x) = 0, \tag{2}$$

the solution to which is sought in the following form:

$$u_k(x) = A_k \sin(\lambda_k x) + B_k \cos(\lambda_k x). \tag{3}$$

In the equations (2) and (3)  $\lambda_k^2 = \frac{p^2 \rho_k}{G_k}$ .

Taking into account the boundary conditions and the conditions of equality of shear deformations and stresses at the levels of the planes of separation of layers, we obtain a system of homogeneous equations with respect to  $n$  unknown coefficients  $A_k$  and  $B_k$ . Having set the determinant of the system equal to zero, we find the frequencies of free vibrations  $p_k$ .

A transverse seismic wave of the SH type emanating vertically from the bedrock is considered as a stationary random function  $\tilde{f}(t)$  with the given spectral density  $S_f(\omega)$ . The solution to the probabilistic problem of vibrations of a multilayer system during the propagation of a seismic wave in a viscoelastic medium is carried out through the spectral representation method. For this purpose, the random function of seismic accelerations  $\tilde{f}(t)$  is defined in the following form:

$$\tilde{f}(t) = \sum_{k=-\infty}^{\infty} \tilde{Z}_k e^{i\omega_k t}, \quad (4)$$

where:  $\tilde{Z}_k$  – are uncorrelated random amplitudes, the sum of the variances of which is equal to the variance of the random function  $\tilde{f}(t)$

$$D_f = \sum_{k=-\infty}^{\infty} D_{Z_k}, \quad (5)$$

The response of the linear stationary system  $\tilde{F}(t)$  to the action (4) is also represented in the form of a spectral decomposition

$$\tilde{F}(t) = \sum_{k=-\infty}^{\infty} \tilde{Z}_k h(i\omega_k) e^{i\omega_k t}, \quad (6)$$

where  $h(i\omega_k)$  – is the amplitude-frequency characteristic (AFC) of the system.

The spectral density  $S_F(\omega)$  of the output random function  $\tilde{F}(t)$  of transverse vibrations of a multilayer system is equal to:

$$S_F(\omega) = |h(i\omega_k)|^2 S_f(\omega). \quad (7)$$

The equations of the multilayer system vibrations with the account for damping under the action of the harmonic load  $\epsilon i \omega t$  have the following form:

$$\begin{aligned} F_0(x; t) &= U_0 e^{i\omega(t+\frac{x_1}{v_0})} e^{-ic_0 x} + D_0 e^{i\omega(t+\frac{x_0}{v_0})} e^{-ic_0 x}, \text{ где: } x_1 = x_0 - H_0; \\ F_1(x; t) &= U_1 e^{i\omega(t+\frac{x_2}{v_1})} e^{-ic_1 x} + D_1 e^{i\omega(t+\frac{x_1}{v_1})} e^{-ic_1 x}, \text{ где: } x_2 = x_1 - H_1; \\ &\dots \dots \dots \\ F_i(x; t) &= U_j e^{i\omega(t+\frac{x_{j+1}}{v_j})} e^{-ic_j x} + D_j e^{i\omega(t+\frac{x_j}{v_j})} e^{-ic_j x}, \text{ где: } x_{j+1} = x_j - H_i; \\ &\dots \dots \dots \\ F(x; t) &= U_n e^{i\omega(t+\frac{x_{n+1}}{v_n})} e^{-ic_n x} + D_n e^{i\omega(t+\frac{x_n}{v_n})} e^{-ic_n x}, \text{ где: } x_{n+1} = x_n - H_n; \\ F_{n+1}(x; t) &= U_{n+1} e^{i\omega(t+\frac{x_{n+1}}{v_{n+1}})} e^{-ic_{n+1} x} + D_{n+1} e^{i\omega(t+\frac{x_{n+1}}{v_{n+1}})} e^{-ic_{n+1} x}, \text{ где: } x_{n+1} = 0 \end{aligned} \quad (8)$$

The following notations are adopted in the equations (8):

$U_j$  – is the wave leaving the  $j+1$ th layer and entering the  $j$ th layer and propagating upward;

$D_j$  – is the wave reflected from the lower boundary of the  $j-1$ th layer, propagating downwards in the  $j$ th layer;

$v_j$  – is the propagation velocity of transverse seismic waves in the  $j$ th layer;

$c_i = c_{1j} - ic_{2j}$  – is the complex wave number.

Taking into account the boundary conditions, we obtain a system of linear equations  $AX = B$  with respect to the unknown amplitudes  $U_0, \dots, U_{n-1}, D_0, \dots, D_n$ . for each value of the frequency  $\omega$ . The Gaussian algorithm is used for the solution.

We obtain the equations of stationary vibrations of the system:

$$\begin{aligned}
 F_0(\omega, x_0) &= U_0(\omega) e^{i\frac{\omega}{v_0}(x_0-H_0-c_{10}(\omega)x_0)} e^{-\frac{\omega}{v_0}c_{20}(\omega)x_0} + D_0(\omega) e^{-i\frac{\omega}{v_0}(x_0+c_{10}(\omega)x_0)} e^{\frac{\omega}{v_0}c_{20}(\omega)x_0}; \\
 F_1(\omega, x_1) &= U_1(\omega) e^{i\frac{\omega}{v_1}(x_1-H_1-c_{11}(\omega)x_1)} e^{-\frac{\omega}{v_1}c_{21}(\omega)x_1} + D_1(\omega) e^{-i\frac{\omega}{v_1}(x_1+c_{11}(\omega)x_1)} e^{\frac{\omega}{v_1}c_{21}(\omega)x_1}; \\
 &\dots\dots\dots \\
 F_n(\omega, x_n) &= U_n(\omega) e^{i\frac{\omega}{v_n}(x_n-H_n-c_{1n}(\omega)x_n)} e^{-\frac{\omega}{v_n}c_{2n}(\omega)x_n} + D_n(\omega) e^{-i\frac{\omega}{v_n}(x_n+c_{1n}(\omega)x_n)} e^{\frac{\omega}{v_n}c_{2n}(\omega)x_n}; \\
 F_{n+1}(\omega, x_{n+1}) &= F_{n+1}(\omega, 0) = 1 + D_{n+1}(\omega) \cdot 1
 \end{aligned} \tag{9}$$

The equations (9) allow obtaining the amplitude-frequency characteristics for each *j*th homogeneous layer

$$h_j(\omega, x_j) = \frac{F_i(\omega, x_j)}{F_{j+1}(\omega, 0)}, \quad x_j=0, \dots, H_j; \quad j=0, 1, \dots, n, \tag{10}$$

And also for the “soil - structure” system as a whole

$$h_s(\omega, x) = \frac{F(\omega, x)}{F_{n+1}(\omega, 0)}, \quad x=0, \dots, H; \quad H=H_0 + H_1 + \dots + H_n, \tag{11}$$

where  $F(\omega, x)$  – is the amplitude of vibrations of the entire packet of layers,  $F_{n+1}(\omega, 0)$  – is the amplitude of the wave leaving the bedrock and entering the surface layers of the soil bed.

The analysis was performed in the MathCAD environment.

### 3. RESULTS AND DISCUSSION

Verification of the layered “soil – structure” model

An investigation was conducted on the possibility of using a layered medium model for the purpose of analysis of the "soil – structure" system for seismic load. Obtained through the use of layered and platform models, the results of comparison of the dynamic characteristics of the building with account for the soil bed flexibility are given below.

A three-storey shear wall-frame reinforced concrete building resting on a two-layer soil bed is considered. The load-bearing structural elements are columns, shear walls, beams and floor slabs. The main geometric characteristics are as follows: building height of 11 m; floor height of 3.5 m; length  $L_x= 21$  m; width  $L_y$  of 19.4 m (Fig. 2). The type of foundation is 1.5×1.5m strip foundation with the laying depth of 1.5 m.

The layered model is formed by the three layers: layer 0 corresponds to the building in the form of a shear member with equivalent characteristics, layers 1 and 2 are the soil bed, layer 3 is the bedrock. The characteristics of the system are shown in Table 1.

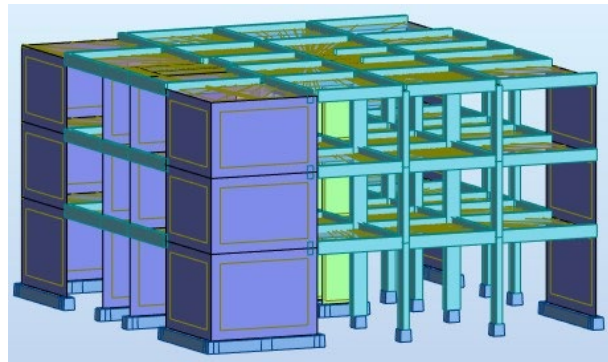


Fig. 2. Analytical scheme of the building.

**Table 1.** Design characteristics of the layered model.

№ of the layer	Thickness H, m	Density $\rho$ , t*s2/m4	Velocity of transverse wave propagation $v$ , m/s
0	11	0.0146	1200
1	10	0.20	570
2	20	0.18	200
3	$\infty$	0.3	1500

For the purpose of verification, we use a platform model with equivalent horizontal stiffness of the soil bed [2].

$$K_x = 2(1 + \mu)G_{sr}\beta_x \sqrt{L_x L_y} = 5.321 \cdot 10^6 \text{ t/m}$$

The averaged characteristics of the two-layer soil bed:

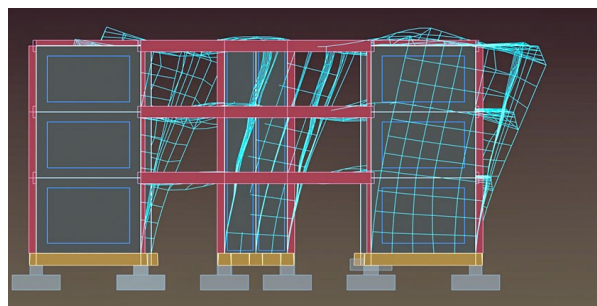
- the average velocity of wave propagation through soil  $v_{sr}=678.57 \text{ m/s}$ ;
  - the average density of the soil  $\rho_{sr}=0.207 \text{ t*s2/m4}$ ;
  - the average shear stiffness of the soil bed  $G_{sr} = 9.52 \cdot 10^4 \text{ t/m2}$ .
- Poisson's ratio  $\mu=0.4$ ;  $\beta_x = 1$ .

The building analysis taking into account the soil bed flexibility is carried out applying the “Robot Structural Analysis” software (Table 2, Fig. 3-4). The following FE were used to model the frame:

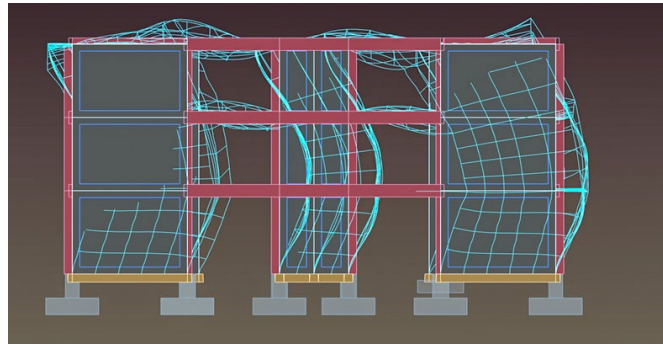
- “frame” with two nodes, each of which has 6 degrees of freedom for beams and column,
- rectangular shell element “shell” for shear walls and floors.

**Таблица 2.** Frequencies of free vibrations of a building on a flexible soil bed.

Loading/ mode	Frequency [Hz]	Period [s]	Cumulative modal mass UX [%]	Cumulative modal mass UY [%]	Cumulative modal mass UZ [%]	Modal mass UX [%]
4/1	5.21	0.19	70.36	0.04	0.0	70.36
4/2	5.35	0.19	70.40	75.38	0.0	0.04
4/3	8.44	0.12	76.23	75.38	0.0	5.82
4/4	19.11	0.05	96.63	75.53	0.0	20.40
4/5	19.32	0.05	96.77	96.29	0.0	0.14
4/6	25.47	0.04	96.79	96.40	0.0	0.02
4/7	26.61	0.04	96.88	96.42	0.0	0.09
4/8	27.91	0.04	96.89	96.51	0.0	0.01
4/9	28.46	0.04	96.90	96.53	0.0	0.01
4/10	28.80	0.03	96.90	96.79	0.0	0.00



**Fig. 3.** The first fundamental mode of translational free vibrations, frequency 5.21 Hz.



**Fig. 4.** The second fundamental mode of translational free vibrations, frequency 19.11 Hz.

Taking into account different degrees of detail of the analytical models, a comparison of the natural frequencies of the “soil – structure” system shows satisfactory results (Table 3). A larger discrepancy in the 1st frequency is due to various conditions of fixing. Thus, the spatial FE model is fixed at the level of the foundation laying, and the layered model is fixed at the depth of 30 m.

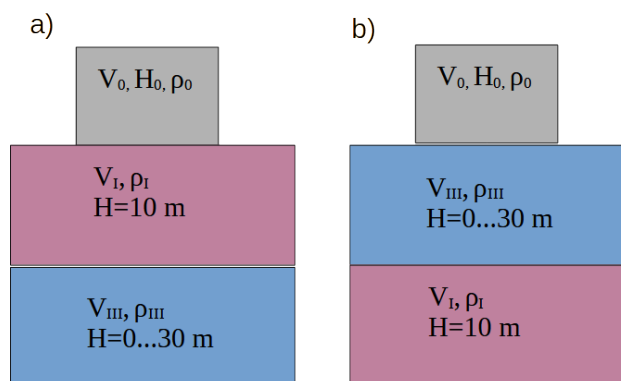
**Table 3.** Comparison of the fundamental frequencies of free vibrations of the “soil – structure” system between the two analytical models.

No.	Platform model		Layered model	Error, %
	Frequency $f$ , Hz	Circular frequency $\omega$ , s <sup>-1</sup>	Circular frequency $\omega$ , s <sup>-1</sup>	
1	5.21	32.74	35.70	8.3
2	19.11	120.07	122.60	2.1

Investigation of the influence of weak layer thickness on the distribution of the resonant frequencies of “two-layer soil bed-structure” system. The soil bed is composed of the two types of soil: stiff soil of class I in terms of seismic properties and soft soil of class III according to the Construction Regulations SP 14.13330 (Table 4). The order of occurrence of the layers is shown in Fig. 5.

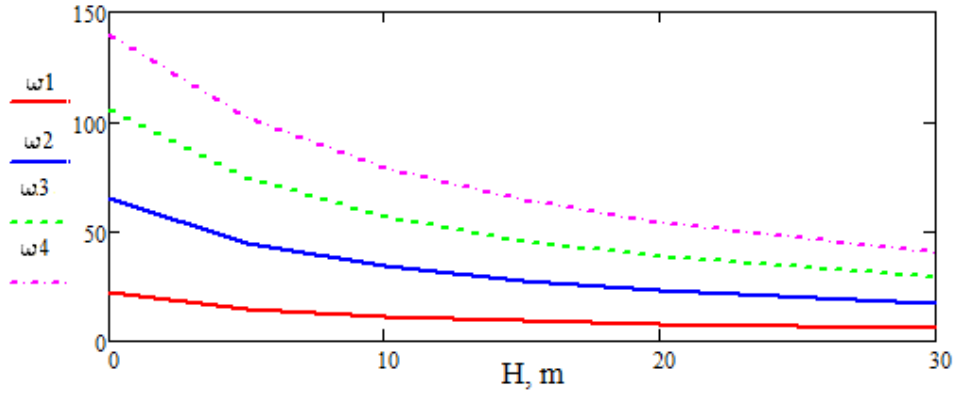
**Table 4.** Characteristics of the layers.

Name of layer	$H$ , m (scheme a)	$H$ , m (scheme b)	$\rho$ , t·s <sup>2</sup> /m <sup>4</sup>	$V_s$ , m/s
Layer №0 “Structure”	11	11	0.0146	1200
Stiff soil of class I	10	0 ... 30	0.21	800
Soft soil of class III	0 ... 30	10	0.18	150

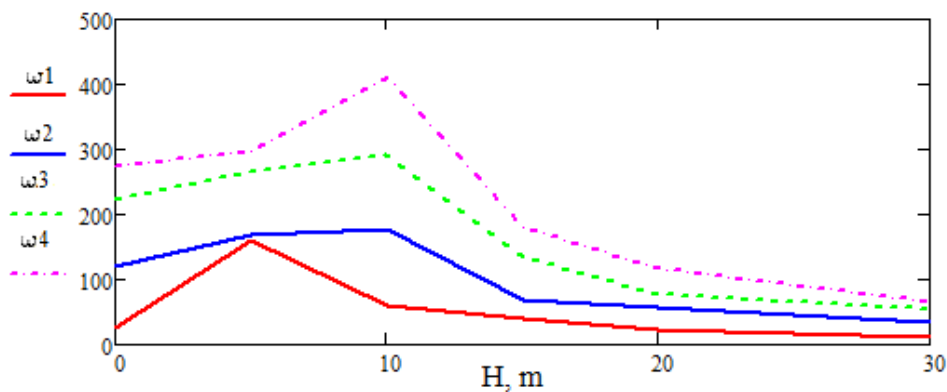


**Fig. 5.** Schemes of the layers occurrence.





**Fig. 6.** Dependence of the resonant frequencies of the system on the thickness of buried weak layer, scheme a).



**Fig. 7.** Dependence of the resonant frequencies of the system on the thickness of near-surface weak layer, scheme b).

As it can be seen from the graphs shown in Fig. 6-7, with an increase in the thickness of the layer with low velocity of transverse waves propagation, the resonant frequencies of the system crowd together and shift towards lower values.

Investigation of the response of the system to seismic load

The results of the seismic analysis of the “two-layer soil bed - structure” system are given below. The seismic acceleration of the soil bed is modelled as a stationary random process with zero mathematical expectation and the given spectral density

$$S_f(\omega) = D_f \frac{2\alpha}{\pi} \left( \frac{m^2 + \omega^2}{m^2 + 2\alpha\omega^2 + \omega^4} \right), \quad (12)$$

where:  $m^2 = \alpha^2 + \beta^2$ ;  $a^2 = \alpha^2 - \beta^2$ ;  $D_f$  – is the seismic acceleration dispersion,

$\alpha$  – is the spectrum bandwidth parameter,  $\beta$  – is the carrier frequency of the seismic acceleration.

Through solving the problem of analysis of a multilayer system for the action of seismic load, we find the following response parameters: the amplitude-frequency characteristic of the system  $|h_s(\omega, x)|$ , the spectral density of vibration acceleration  $S_F(\omega, x)$ , dynamic coefficient  $\beta(x)$  ( $x=0, \dots, H_0+H_1+H_2$ ). The average value of the thickness of the soil layers is accepted as  $H_1=H_2=15$ m (tables 5-6).

**Table 5.** Response of a layered system to seismic impact.

Number of layer	$H, m$	Upper weak layer		Lower weak layer	
		$\rho, t \cdot s^2/m^4$	$V_s, m/s$	$\rho, t \cdot s^2/m^4$	$V_s, m/s$
Layer №0 «Structure»	11	0.0146	1200	0.0146	1200
Layer №1	15	0.18	200	0.2	800
Layer №2	15	0.2	800	0.18	200
Layer №3 «Bedrock»	$\infty$	3.0	1500	3.0	1500
Upper weak layer		Lower weak layer			
Amplitude-frequency characteristics $ h_s(\omega, H_0) $					
Spectral density of vibration acceleration $S_F(\omega, H_0)$					
Dynamic coefficient $\beta(H_0)$					
<p style="text-align: center;"><math>\beta(H_0) = 2.85</math></p>			<p style="text-align: center;"><math>\beta(H_0) = 1.74</math></p>		

**Table 6.** Comparison of the dynamic response of a system with a two-layer soil bed with a 30-metre lower layer and a system with the reduced characteristics of the soil bed.

Number of soil bed layer	$H, m$	Lower weak layer		Reduced characteristics of two-layer soil bed	
		$\rho, t \cdot s^2/m^4$	$V_s, m/s$	$\rho, t \cdot s^2/m^4$	$V_s, m/s$
Layer №1	15	0.20	800	0.207	678.57
Layer №2	30	0.18	200		
Amplitude-frequency characteristics $ h_s(\omega, H_0) $					
Spectral density of vibration acceleration $S_F(\omega, H_0)$ , parameters of the spectral density of seismic load $\alpha=6 s^{-1}, \beta=19 s^{-1}$					
Dynamic coefficient $\beta(H_0)$					
<p style="text-align: center;"><math>\beta(H_0) = 2.344</math></p>			<p style="text-align: center;"><math>\beta(H_0) = 2.48</math></p>		
Spectral density of vibration acceleration $S_F(\omega, H_0)$ , parameters of the spectral density of seismic load $\alpha=3.21 s^{-1}, \beta=9 s^{-1}$					
Dynamic coefficient $\beta(H_0)$					
<p style="text-align: center;"><math>\beta(H_0) = 2.53</math></p>			<p style="text-align: center;"><math>\beta(H_0) = 1.32</math></p>		

At average parameters of the spectral density of seismic acceleration  $\alpha=6 \text{ s}^{-1}$ ,  $\beta=19 \text{ s}^{-1}$  the dynamic coefficients of the layered and reduced models practically coincide. For low-frequency earthquakes with the spectral density parameters of  $\alpha=3.21 \text{ s}^{-1}$ ,  $\beta=9 \text{ s}^{-1}$  (the horizontal component of the 1976 Gazli earthquake), the values of the dynamic coefficients differ by almost 2 times.

The carrier frequencies of the amplitude-frequency characteristic of the system are determined by the resonant properties of layer № 2. Moreover, it should be noted that they differ from the resonant frequencies of a freely vibrating layer with the same characteristics (Table 7).

**Table 7.** Comparison of the resonant frequencies of layer №2 and the frequencies of the free vibrations of a homogeneous layer with similar characteristics.

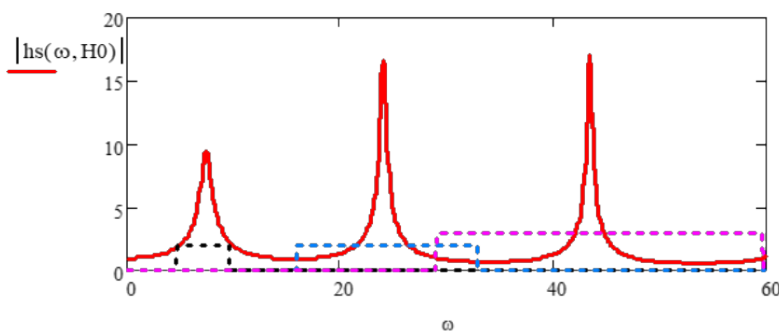
Number of frequency	Frequencies of layer №2 in two-layer system	Frequencies of layer №2 as of a homogeneous medium	Percentage difference
1	7.4	10.47	34.36
2	24	31.41	26.75
3	43.4	52.36	18.71

For the reduced layer, we obtain sparser resonant frequencies.

Probabilistic analysis of the frequency distribution of free vibrations of “soil - structure” system

According to the Construction Regulations SP 23.13330.2017, during an analysis of a “soil - structure” system for seismic resistance, the statistical variability of the dynamic parameters of soil bed should be taken into account. The variation coefficients values of the design parameters of various types of soils are as follows: density –  $f_{\rho}=0.05-0.1$ ; velocity of transverse wave propagation –  $f_v=0.2-0.35$ ; shear modulus –  $f_G=0.2-0.66$ . The greatest variability is observed in soft soils with low velocity of seismic waves propagation.

The results of the computation of the amplitude-frequency characteristics of the layered “two-layer soil bed - structure” system (Table 6) taking into account the random variability of the transverse waves velocity within each layer are given below. Soil density is considered as a deterministic value due to its low variability. The law of random variables distribution  $V_{si}$  is assumed to be normal. The variation coefficients values of the velocities of transverse waves of soil layers are the following:  $f_{v1}=0.1$ ;  $f_{v2}=0.33$ . We find the intervals of possible values of the resonant frequencies of the system through the Monte Carlo method.



**Fig. 8.** Intervals of the variability of the resonant frequencies of the system within one standard.

The following values of the statistical characteristics were obtained for the first four frequencies:

– mathematical expectation:  $\omega_1=6.9 \text{ s}^{-1}$ ;  $\omega_2=23.8 \text{ s}^{-1}$ ;  $\omega_3=43.2 \text{ s}^{-1}$ ;  $\omega_4=6.9 \text{ s}^{-1}$ ;

– variation coefficient:  $f_{\omega_1}=0.35$ ;  $f_{\omega_2}=0.35$ ;  $f_{\omega_3}=0.34$ ;  $f_{\omega_4}=0.34$ .

For a system containing a low-velocity layer, the intervals of possible values of adjacent frequencies overlap (Fig. 8).

## 5. CONCLUSIONS

A wave model of layered medium was suggested for the purpose of investigation of the special features of the interaction between a structure and a multilayer soil bed under seismic loads. Taking into account the fact that the dimensions of structural inhomogeneities of buildings are, as a rule, much smaller than the length of a seismic wave, a structure is considered as an element of a layered system in the form of a shear member with equivalent dynamic characteristics. The solution to the problem is based on the assumption of vertical propagation of SH-type waves. Seismic acceleration of the soil bed is modeled as a stationary random process.

An agreement between the frequencies of the free translational vibrations of the "soil - structure" system obtained by means of the layered and platform models was found to be satisfactory for the purposes of practical investigations. The percentage difference was as follows: 8.3% for the first frequency, 2.1% for the second frequency.

The influence of physical, mechanical and geometric parameters of a multilayer soil bed on the pattern of resonant frequencies distribution, response spectra and dynamic coefficients of the "soil - structure" system was investigated.

It was revealed that the spectrum of the natural frequencies of the "multilayer soil bed - structure" system becomes more crowded and shifts towards lower values with an increase in the thickness of the weak layer, regardless of its location. For the near-surface layer, the frequencies get lower by an average of 3.6 times; while for the buried layer they get lower by 5.2 - 14 times. In this case, if the near-surface layer is composed of stiffer soil, then a uniform, close to linear, change in the configuration of resonant frequencies is observed with an increase in the thickness of the buried weak layer (figure 7). For the near-surface weak layer, this dependence becomes significantly nonlinear (figure 8).

A comparison of the dynamic response of the two systems to seismic load was carried out: 1) "two-layer soil bed - structure" with different characteristics of the layers, 2) "homogeneous soil bed - structure" with equivalent characteristics. The averaged graphs of the amplitude-frequency characteristics and spectral density of system 2) differ quantitatively and qualitatively from the graphs of system 1). In the course of the analysis for seismic load with averaged spectral density parameters  $\alpha=6$  s<sup>-1</sup>,  $\beta=19$  s<sup>-1</sup>, practically identical dynamic coefficients were obtained for the two systems: 2.34 and 2.48, respectively. For low-frequency earthquakes (Gazli, 1976), system 2) shows almost 2 times lower values of the dynamic coefficients: 2.53 and 1.32. It should be noted that the carrier frequencies of the amplitude-frequency characteristics of the system are determined by the resonant properties of the weakest layer. The results of the investigation prove the necessity to take into account the multilayer arrangement of soil bed when designing structures with respect to their seismic safety.

The influence of statistical variability in the velocity of transverse waves propagation within each of the layers of a two-layer soil bed on the variations in the resonant frequencies of the system was investigated. The following values of the velocity variation coefficients were adopted:  $f_{v1}=0.1$ ;  $f_{v2}=0.33$ . The values of the variation coefficients of the first four frequencies of the system are 0.34-0.35. Random variability in the velocity of seismic waves propagation implies that intervals of the possible values of resonant frequencies are considered instead of their particular values. In this case, adjacent intervals can overlap each other (figure 9).

Investigating wave processes in the "multilayer soil bed - structure" system plays a key role in ensuring the safety of structures under seismic impacts. The obtained results will help to improve the prediction of the dynamics of structures and optimize the design with account for multilayer soil beds.

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